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Kadomtsev-Petviashvili (KP) Burgers equation in a dusty plasmas with non-adiabatic dust charge fluctuation

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Abstract. The nonlinear dust acoustic waves in a dusty plasmas with the combined effects of non-adiabatic dust charge fluctuation and higher-order transverse perturbation are studied. Using the perturbation method, a Kadomtsev-Petviashvili (KP) Burgers equation that governing the dust acoustic waves is deduced for the first time. A particular solution of this KP Burgers equation is also obtained. It is show that the dust acoustic shock waves can exist in the KP Burgers equation.

PACS. 52.35.Sb Solitons; BGK modes – 52.35.Mw Nonlinear phenomena: waves, and nonlinear wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Introduction

Recently, nonlinear wave phenomenon in dusty plasmas have been received considerable attention. Theoretical studies indicate [1–6] that the presence of the charged dust grains in a plasma would modify the collective behavior of a plasma, as well as excite new modes. In reality, the charge on the dust grain varies both with space and time due to electron and ion currents flowing into or out of the dust grain, as well as other processes such as photoemission of electrons. Those cause the dust charge fluctuations. So, the dust charge becomes a new dynamic variable and its dynamic nature would be important for studying the behavior of dusty plasmas. Under the assumption of adiabatic ($\tau_{\rm ch}/\tau_{\rm d} = 0$) and non-adiabatic ($\tau_{\rm ch}/\tau_{\rm d}$ is small but finite) dust charge variation, where $\tau_{\rm ch}$ is the charging time scale and $\tau_{\rm d}$ is the hydrodynamical time scale, recent studies indicate [7–11] that the dust charge fluctuation would effects the wave mode properties and led to some new aspects of dusty plasmas. While the viscosity, particle reflection, interparticle collisions, and Landau damping can led to the energy dissipation, the nonadiabaticity of the dust charge fluctuation provides an alternate physical mechanism causing the dissipation. Assuming small but finite $\tau_{\rm ch}/\tau_{\rm d}$, theoretical studies show [12,13] that the dust charge fluctuation can led to a strong dissipation of the wave because of a phase difference between the dust charge fluctuation and the wave. The nonlinear investigations indicate [10,11] that the nonlinear dust ion acoustic wave (dust acoustic wave) is governed by the Korteweg-de Vries (KdV) Burgers equation and the dust ion acoustic (dust acoustic) shock wave is generated due to the dissipation caused by the non-adiabatic charge variation of the dust particles. But all previous investigations for the dust charge fluctuation effects are focused on the one-dimensional case. However, a purely one-dimensional picture cannot explain the observed wave phenomena in the low altitude and higher altitude auroral regions. The waves structure and stability in this higher dimensional system will be modified because the anisotropy is introduced into the system. For example, at least some transverse perturbations will always exist in the higher dimensional system even if the system is un-magnetized. Therefore, in present paper, we study the nonlinear dust acoustic waves under the higher order transverse perturbation by incorporating the dust charge fluctuation effect. Using the perturbation method, a Kadomtsev-Petviashvili (KP) Burgers equation governing the dust acoustic wave is deduced. The exact wave frame solution of this KP Burgers equation is obtained. It is show that the dust acoustic shock wave can exist in the KP Burgers equation.

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2 Governing equations

Consider the dust acoustic waves propagating in a collisionless un-magnetized plasma whose constituents are Bolzmann distributed electrons, ions, and massive high negatively charged warm adiabatic dust grains. The non-adiabatic dust charge fluctuations are also considered. Then the dust dynamics can be described by the following two-dimensional set of continuity, momentum, and Poisson's equations:

$$\frac{\partial n_{\rm d}}{\partial t}+\frac{\partial(n_{\rm d}u_{\rm d})}{\partial x}+\frac{\partial(n_{\rm d}v_{\rm d})}{\partial y}=0 \tag{1}$$

$$\frac{\partial u_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial u_{\rm d}}{\partial x}+v_{\rm d}\frac{\partial u_{\rm d}}{\partial y}=-(q-1)\frac{\partial\phi}{\partial x}-3\sigma_{\rm d}n_{\rm d}\frac{\partial n_{\rm d}}{\partial x} \tag{2}$$

$$\frac{\partial v_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial v_{\rm d}}{\partial x}+v_{\rm d}\frac{\partial v_{\rm d}}{\partial y}=-(q-1)\frac{\partial\phi}{\partial y}-3\sigma_{\rm d}n_{\rm d}\frac{\partial n_{\rm d}}{\partial y} \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\frac{1}{1-\delta}{\rm e}^{\phi}-\frac{\delta}{1-\delta}{\rm e}^{-\phi/\sigma_{\rm i}}-n_{\rm d}(q-1) \tag{4}$$

where $\sigma_{\rm d}=T_{\rm d}/z_{\rm d}T_{\rm e}$, $\delta=n_{\rm i0}/n_{\rm e0}$, $\sigma_{\rm i}=T_{\rm i}/T_{\rm e}$. The variables are normalized as follows: $t=\omega_{\rm pd}t'$, $x=x'/\lambda_{\rm D}$, $u_{\rm d}=u'_{\rm d}/c_{\rm d}$, $\phi={\rm e}\phi'/T_{\rm e}$, $(n_{\rm e},n_{\rm i})=(n'_{\rm e},n'_{\rm i})/z_{\rm d}n_{\rm d0}$, $n_{\rm d}=n'_{\rm d}/n_{\rm d0}$. Here $\omega_{\rm pd}=(z_{\rm d}^2{\rm e}^2n_{\rm d0}/4\pi m_{\rm d})^{1/2}$, $\lambda_D=(4\pi T_{\rm e}/z_{\rm d}n_{\rm d0}{\rm e}^2)^{1/2}$, $c_{\rm d}=(z_{\rm d}T_{\rm e}/m_{\rm d})^{1/2}$ are the dusty plasmas frequency, dust Debye length, and dust acoustic velocity, respectively. The dust charge $Q=-z_{\rm d}e+q$, where q is the fluctuating dust charge, becomes $(q-1)$, normalized in units of the equilibrium dust charge $z_{\rm d}e$. The normalized charge variable q is determined by the following normalized orbital motion limited charge current balance equation [12,14]

$$\frac{\tau_{\rm ch}}{\tau_{\rm d}}\left(\frac{\partial q}{\partial t}+u_{\rm d}\frac{\partial q}{\partial x}+v_{\rm d}\frac{\partial q}{\partial y}\right)=\frac{\tau_{\rm ch}}{z_{\rm d}e}(I_{\rm e}+I_{\rm i}) \tag{5}$$

where $I_{\rm e}$ and $I_{\rm i}$ are the electron and ion current, respectively. The normalized expressions for the electron and ion currents for spherical dust grains with radius a are

$$I_{\rm e}=-\pi a^2e\left(\frac{8T_{\rm e}}{\pi m_{\rm e}}\right)^{1/2}n_{\rm e0}{\rm exp}(\phi){\rm exp}[z(q-1)] \tag{6}$$

$$I_{\rm i}=\pi a^2e\left(\frac{8T_{\rm i}}{\pi m_{\rm i}}\right)^{1/2}n_{\rm i0}{\rm exp}(-\phi/\sigma_{\rm i})\left[\left(1+\frac{z}{\sigma_{\rm i}}\right)-\frac{z}{\sigma_{\rm i}}q\right] \tag{7}$$

where $z=z_{\rm d}e^2/aT_{\rm e}$. $\tau_{\rm d}\approx\omega_{\rm pd}^{-1}$ is the dust oscillation time scale and

$$\tau_{\rm ch}=\left[\frac{a}{(2\pi)^{1/2}}\frac{\omega_{\rm pi}^2}{V_{\rm thi}}(1+z+\sigma)\right]^{-1}$$

is the charging time scale. Here $V_{\rm thi}$ is the ion thermal velocity. Equations (1–7) govern the dynamics of fully nonlinear dust acoustic waves in a warm dusty plasmas.

3 Derivation of the KP-Burgers equation

In order to investigate the nonlinear propagation of dust acoustic wave in the plasma, we employ the perturbation technique to obtain the KP-Burgers equation. The independent variables are stretched as $\xi=\epsilon^{1/2}(x-v_0t)$, $\eta=\epsilon y$, and $\tau=\epsilon^{3/2}t$, where ϵ is a small parameter and v_0 is the velocity of dust acoustic wave. The dependent variables are expanded as

$$n_{\rm d}=1+\epsilon n_1+\epsilon^2n_2+...,\tag{8}$$

$$u_{\rm d}=\epsilon u_1+\epsilon^2u_2+...,\tag{9}$$

$$v_{\rm d}=\epsilon^{3/2}v_1+\epsilon^{5/2}v_2+...,\tag{10}$$

$$\phi=\epsilon\phi_1+\epsilon^2\phi_2+...,\tag{11}$$

$$q=\epsilon q_1+\epsilon^2q_2+...\tag{12}$$

To make the nonlinear perturbation consistent, we assume that $\tau_{\rm ch}/\tau_{\rm d}$ is small and is proportional to $\epsilon^{1/2}$. Thus we take

$$\frac{\tau_{\rm ch}}{\tau_{\rm d}}=\mu\epsilon^{1/2}\tag{13}$$

where μ is a finite quantity of the order of unity. Substituting the expression (8–13) into equations (1–7) and collecting the terms in the different powers of ϵ, for the lowest order equation, we can obtain the following relations:

$$v_0n_1=u_1,\tag{14}$$

$$v_0u_1=3\sigma_{\rm d}n_1-\phi_1,\tag{15}$$

$$\left[\frac{1}{1-\delta}+\frac{\delta}{\sigma_{\rm i}(1-\delta)}\right]\phi_1-q_1+n_1=0\tag{16}$$

$$q_1=-\beta_1\phi_1.\tag{17}$$

From equations (14–17), the wave velocity v_0 is also obtained as

$$v_0^2=3\sigma_{\rm d}+\frac{1}{\beta_1+\frac{1}{1-\delta}+\frac{\delta}{\sigma_i(1-\delta)}}\tag{18}$$

where $\delta=n_{\rm i0}/n_{\rm e0}$ and

$$\beta_1=\frac{(z+\sigma_{\rm i})[(1-z)(1+\sigma_{\rm i})+z^2/2]}{z\sigma_{\rm i}[(1-z)(1+z+\sigma_{\rm i})+z^2/2]}\cdot\tag{19}$$

The variable β_1 stand for the dust charge fluctuation effect. We can also obtain the next higher order y component of the momentum equation

$$v_0\frac{\partial v_1}{\partial\xi}=3\sigma_{\rm d}\frac{\partial n_1}{\partial\eta}-\frac{\partial\phi_1}{\partial\eta}\cdot\tag{20}$$

To next higher order in ϵ, from the continuity equation, the x-component of the momentum equation, Poisson's equation, and charge balance equation, we obtain

$$\frac{\partial n_1}{\partial \tau} - v_0\frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial n_1 u_1}{\partial \xi} + \frac{\partial v_1}{\partial \eta} = 0 \quad (21)$$

$$\frac{\partial u_1}{\partial \tau} - v_0\frac{\partial u_2}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} + 3\sigma_d\frac{\partial n_2}{\partial \xi} + u_1\frac{\partial u_1}{\partial \xi} + q_1\frac{\partial \phi_1}{\partial \xi} + 3\sigma_d n_1\frac{\partial n_1}{\partial \xi} = 0 \quad (22)$$

$$n_2 + \left[\frac{1}{1-\delta} + \frac{\delta}{\sigma_{\rm i}(1-\delta)}\right]\phi_2 - q_2 + \frac{1}{2}\left[\frac{1}{1-\delta} - \frac{\delta}{\sigma_{\rm i}^2(1-\delta)}\right]\phi_1^2 - n_1 q_1 - \frac{\partial^2\phi_1}{\partial \xi^2} = 0 \quad (23)$$

$$\beta_2\phi_2 + \frac{\beta_2}{\beta_1}q_2 + \beta_3\phi_1 q_1 + \beta_4\phi_1^2 + \beta_5 q_1^2 - \mu v_0\frac{\partial q_1}{\partial \xi} = 0 \quad (24)$$

where

$$\beta_2 = \frac{(z+\sigma_{\rm i})[(1-z)(1+\sigma_{\rm i}) + z^2/2]}{z\sigma_{\rm i}(1-z+z^2/2)(1+z+\sigma_{\rm i})}, \quad (25)$$

$$\beta_3 = \frac{1-z+z^2/2-\sigma_{\rm i}(\sigma_{\rm i}+z)}{(1+z+\sigma_{\rm i})(1-z+z^2/2)\sigma_{\rm i}} \quad (26)$$

$$\beta_4 = -\frac{(1-z+z^2/2-\sigma_{\rm i}^2)(\sigma_{\rm i}+z)}{2z(1+z+\sigma_{\rm i})(1-z+z^2/2)\sigma_{\rm i}^2} \quad (27)$$

$$\beta_5 = \frac{z(\sigma_{\rm i}+z)}{2(1+z+\sigma_{\rm i})(1-z+z^2/2)}.$$

Substituting the above derived expressions of equations (14–17), and equation (20) into equations (21–24) and eliminating the term n_2, u_2, ϕ_2, and q_2, then we deduce the following KP-Burgers equation for dust acoustic wave in the plasma with dust charge fluctuation effect

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3\phi}{\partial \xi^3} - C\frac{\partial^2\phi}{\partial \xi^2}\right] + D\frac{\partial^2\phi}{\partial \eta^2} = 0 \quad (28)$$

where $\phi \equiv \phi_1$ and

$$A = -\frac{3(v_0^2+\sigma_{\rm d})}{2v_0(v_0^2-3\sigma_{\rm d})} + \frac{3\beta_1(v_0^2-3\sigma_{\rm d})}{2v_0} - \frac{(v_0^2-3\sigma_{\rm d})^2}{v_0} \times\left[\frac{1}{2}\left(\frac{1}{1-\delta} - \frac{\delta}{\sigma_{\rm i}^2(1-\delta)}\right) + \frac{\beta_1\beta_4}{\beta_2} - \frac{\beta_1^2\beta_3}{\beta_2} + \frac{\beta_1^3\beta_5}{\beta_2}\right] \quad (29)$$

$$B = \frac{(v_0^2-3\sigma_{\rm d})^2}{2v_0} \quad (30)$$

$$C = \mu\frac{(v_0^2-3\sigma_{\rm d})^2}{2}\frac{\beta_1^2}{\beta_2} \quad (31)$$

$$D = \frac{v_0}{2}. \quad (32)$$

Note that when the Burgers term C (introduced by the dust charge fluctuation) vanish, the KP-Burgers equation (28) reduce to the ordinary KP equation. On the other hand, when the transverse perturbation omitted, the KP-Burgers equation (28) will reduce to the ordinary KdV Burgers equation.

4 Shock wave solution of KP-Burgers equation

For KdV Burgers equation, the wave breaking due to the nonlinearity is balanced by the combined action of dispersion and dissipation, then a dispersive shock wave is generated. Now let we consider the wave propagation in the plasma with both the presence of the non-adiabatic dust charge fluctuation and transverse perturbation effects. On transforming to the wave frame

$$\theta = k_1\xi + k_2\eta - \omega\tau. \quad (33)$$

The KP-Burgers equation (28) reduces to

$$k_1\frac{\rm d}{{\rm d}\theta}\left[-\omega\frac{{\rm d}\phi}{{\rm d}\theta} + Ak_1\phi\frac{{\rm d}\phi}{{\rm d}\theta} + Bk_1^3\frac{{\rm d}^3\phi}{{\rm d}\theta^3} - Ck_1^2\frac{{\rm d}^2\phi}{{\rm d}\theta^2}\right] + Dk_2^2\frac{{\rm d}^2\phi}{{\rm d}\theta^2} = 0 \quad (34)$$

where k_1 and k_2 are the wave numbers in x and y directions. Imposing the boundary conditions for localized perturbation, *viz.* $\phi \to 0, {\rm d}\phi/{\rm d}\theta \to 0, {\rm d}^2\phi/{\rm d}\theta^2 \to 0$, and ${\rm d}^3\phi/{\rm d}\theta^3 \to 0$ at $\theta \to \infty$, equation (34) reduces to

$$Bk_1^4\frac{{\rm d}^2\phi}{{\rm d}\theta^2} - Ck_1^3\frac{{\rm d}\phi}{{\rm d}\theta} + \frac{1}{2}Ak_1^2\phi^2 + (Dk_2^2 - \omega k_1)\phi = 0. \quad (35)$$

This second-order equation can be written as a system of two first-order equations

$$\frac{{\rm d}\phi}{{\rm d}\theta} = \psi \quad (36)$$

$$\frac{{\rm d}\psi}{{\rm d}\theta} = \frac{C}{Bk_1}\psi - \frac{A}{2Bk_1^2}\phi^2 - \frac{1}{Bk_1^4}(Dk_2^2 - \omega k_1)\phi. \quad (37)$$

The equations of the system (36–37) have two fixed points $(\phi_1^*, \psi_1^*) = (0,0)$ and $(\phi_2^*, \psi_2^*) = (-\frac{2(Dk_2^2-\omega k_1)}{Ak_1^2}, 0)$. It is easy to analyze that the second one (ϕ_2^*, ψ_2^*) is a saddle point while the first one (ϕ_1^*, ψ_1^*) is a unstable node or a unstable focus according as

$$C^2 \geq 4B(Dk_2^2 - \omega k_1) \quad (38)$$

or

$$C^2 \leq 4B(Dk_2^2 - \omega k_1) \quad (39)$$

This means that there is a heteroclinic orbit connecting the saddle-node or saddle-focus point in equations (36, 37). On the other hand, shock waves can exist in the system. Therefore, for the saddle-node heteroclinic orbit, equation (35) has the following form particular solution

$$\phi = \frac{m}{[1+{\rm e}^{n(\theta-\theta_0)}]^2}. \quad (40)$$

When substituting equation (40) into equation (35), we can obtain the following relations

$$m = -\frac{12}{25}\frac{C^2}{AB} \quad (41)$$

$$n = -\frac{C}{5Bk_1} \quad (42)$$

$$\frac{Dk_2^2 - \omega k_1}{k_1^2C^2} = \frac{6}{25B}. \quad (43)$$

Hence we get a particular solution of equation (35)

$$\phi = -\frac{3C^2}{25AB}\left[1 - \tanh\frac{C}{10Bk_1}(\theta - \theta_0)\right]^2. \qquad (44)$$

Equation (43) indicates that $\frac{4B(Dk_2^2 - \omega k_1)}{C^2} = \frac{24k_1^2}{25} < 1$, which means that the condition (38) satisfied. Also, it is clear from equation (44) that $\phi \to \phi_1^*$ as $\xi \to +\infty$ and $\phi \to \phi_2^*$ as $\xi \to -\infty$. That is, the particular solution given by equation (44) corresponding to the heteroclinic orbit connecting the saddle-node points. On the other hand, shock wave can exist in the KP-Burgers equation (28).

5 Conclusions

In summary, A KP-Burgers equation describing the dust acoustic waves in dusty plasmas with non-adiabatic dust charge fluctuation is derived by the perturbation method.A shock solution of the KP-Burgers equation is obtained. It is show that one dimensional shock wave can exist even if the system is under the transverse perturbation. The non-adiabatic charge variation of the dust particles would modifies the shock wave properties because the coefficients in KP-Burgers equation are related to the dust charge variation effects.

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